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HOT ACCRETION WITH SATURATED CONDUCTION

KRISTEN MENOU
Department of Astronomy, Columbia University, 550 West 120th Street, New York, NY 10027
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ABSTRACT

Observations of the hot gas surrounding Sgr A* and a few other nearby galactic nuclei imply electron and proton mean free paths comparable to the gas capture radius: hot accretion likely proceeds under weakly-collisional conditions in these systems. As a result, thermal conduction, rather than convection, may be important on all scales and affect the global flow properties. The self-similar ADAF solution of Narayan & Yi (1994) is generalized to include a saturated form of thermal conduction, as is appropriate for the weakly-collisional regime of interest. Conduction provides extra heating and yet it reduces the free-free radiative efficiency of the accretion flow (by potentially large factors). These idealized solutions suggest that thermal conduction may be an important physical ingredient to understand hot accretion onto dim accreting black holes. Conduction could also play a role in reducing the rate at which black holes capture ambient gas and in providing an evaporation mechanism for an underlying cold thin disk.

Subject headings: accretion, accretion disks – conduction – black hole physics – hydrodynamics

1. INTRODUCTION

Over the past decade, X-ray observations have made it increasingly clear that black holes are capable of accreting gas under a variety of distinct configurations. There is convincing evidence for a hot form of accretion occurring at sub-Eddington rates in particular, which contrasts with the classical "cold and thin" accretion disk scenario (Shakura & Sunyaev 1973). Hot accretion appears common in the population of supermassive black holes in galactic nuclei and during the quiescent phases of accretion onto stellar-mass black holes in X-ray transients (e.g. Narayan et al. 1998; Lasota et al. 1996; Di Matteo et al. 2000; Esin et al. 1997, 2001; Menou et al. 1999; see Narayan, Mahadevan & Quataert, 1998; Narayan 2003 for reviews).

Even today, however, the exact nature, structure and properties of these hot accretion flows remain controversial. Inspired by the work of Shapiro, Lightman & Eardley (1976), Narayan & Yi (1994; 1995a,b) derived self-similar ADAF solutions which emphasized the important stabilizing role of radial heat advection (see also Ichimaru 1977; Rees et al. 1982; Abramowicz et al. 1988). Subsequent analytical work on hot accretion flows has emphasized outflows, motivated by a positive Bernoulli constant (ADIOS; Blandford & Begelman 1999), and the potential role of convection (CDAFs; Narayan et al. 2000, Quataert & Gruzinov 2000). Hydrodynamical and MHD numerical simulations have also greatly contributed to the subject by highlighting important dynamical aspects of the problem (Hawley et al. 2001; DeVilliers et al. 2003; Igumenshchev et al. 2003).

The purpose of this *Letter* is to make the case that thermal conduction, which has been a largely neglected ingredient, could affect the global properties of hot accretion flows substantially. In §2, we use existing observational constraints on a few nearby galactic nuclei to argue that hot accretion is likely to proceed under weakly-collisional conditions in these systems, thus implying that thermal conduction could be important. In §3, we extend the original 1-temperature self-similar solutions of Narayan & Yi (1994) with a saturated form of thermal conduction and study the effects of conduction on the flow structure and properties. In §4, we comment on some of the limitations of our work, on possibilities for future work and on potential additional consequences of thermal conduction for hot accretion.

2. OBSERVATIONAL CONSTRAINTS AND COLLISIONALITY

Chandra observations provide tight constraints on the density and temperature of gas at or near the Bondi capture radius in Sgr A* and several other nearby galactic nuclei. These observational constraints have been used before to estimate the rate at which the gas is captured by the black hole in these systems, following Bondi theory (e.g. Loewenstein et al. 2001; Di Matteo et al. 2003; Narayan 2003). Here, we use these same constraints (taken from Loewenstein et al. 2001; Baganoff et al. 2003; Di Matteo et al. 2003; Ho, Terashima & Ulvestad 2003) to calculate mean free paths for the observed gas.[1] Galactic nuclei and the corresponding gas properties on $1''$ scales are listed in Table 1: $n_{1''}$ is the gas number density, $T_{1''}$ is the temperature, R_1 is the $1''$ size-equivalent at the nucleus distance, l_1 is the mean free path for the observed $1''$ conditions and $R_{\rm cap}$ is the capture radius, inferred from the gas temperature and the black hole mass in each nucleus. We have used Spitzer's (1962) expressions for collision times, a standard expression for the thermal speed and a Coulomb logarithm $\ln\Lambda \simeq 20$ to recover the simple mean free path scaling $l \simeq 10^4 (T^2/n)$ cm of Cowie & McKee (1997; valid for both electrons and protons in a 1-temperature gas). We have used this relation to calculate l_1 values in Table 1.

The inferred mean free paths are in the few hundredths to few tenths of the observed $1''$ scales (l_1/R_1 values in Table 1). Assuming no change in the gas properties down to the inferred capture radius, values of $l_1/R_{\rm cap} \gtrsim 0.1$ are deduced in four of the six nuclei. One could also reformulate these scalings by stating that the $1''$ mean free paths are systematically $\gtrsim 10^{4-5} R_{\rm s}$, where $R_{\rm s}$ is the Schwarzschild radius (the typical lengthscale associated with the accreting black holes). These numbers strongly suggest that accretion will proceed under weakly-collisional conditions in these systems.

[1] It should be noted that these constraints are not all equally good, in the sense that *Chandra* has probably resolved the gas capture radius in Sgr A* and M87 but not in the other nuclei.

TABLE 1. OBSERVATIONAL CONSTRAINTS ON COLLISIONALITY

Nucleus	$n_{1''}$ (cm^{-3})	$T_{1''}$ (10^7 K)	R_1 (cm)	l_1/R_1	l_1/R_{cap}
Sgr A*	100	2.3	1.3×10^{17}	0.4	0.4
NGC 1399	0.3	0.9	3.1×10^{20}	0.009	0.02
NGC 4472	0.2	0.9	2.5×10^{20}	0.016	0.07
NGC 4636	0.07	0.7	2.2×10^{20}	0.032	0.6
M87	0.17	0.9	2.7×10^{20}	0.018	0.02
M32	0.07	0.4	1.2×10^{19}	0.2	1.3

The weakly-collisional nature of ADAFs has been noted before (e.g. Mahadevan & Quataert 1997), in the sense of collision times longer than the gas inflow time, but these are model-dependent statements. Direct observational constraints on the gas properties near the capture radius favor a weakly-collisional regime of hot accretion, more or less independently of the exact hot flow structure. Whether the gas adopts an ADAF, CDAF or ADIOS type configuration once it crosses the capture radius, it is expected to become even more weakly-collisional as it approaches the black hole, since the relative mean free path scales as $l/R \propto R^{-3/2-p}$ in these flows with a virial temperature profile and a density profile $\rho \propto R^{-3/2+p}$. It should be noted, however, that tangled magnetic fields provide a way of limiting the weak collisionality of the flow, even though their efficiency at doing so is not well known. We go back this important issue in §4.

3. SELF-SIMILAR SOLUTION WITH SATURATED CONDUCTION

The standard Spitzer formula for thermal conduction applies only to gas well into the collisional ("fluid") regime, with a mean free path $l \ll L$, for any relevant flow scale L. Anticipating temperature variations on local scales $\sim R$ from self-similarity, in the weakly-collisional regime of interest here, with $l \sim R$ or even $l \gg R$, a saturated (or equiv. "flux-limited") thermal conduction formalism must be adopted if one is to avoid unphysically large heat fluxes. We adopt the formulation of Cowie and McKee (1977) and write the saturated conduction flux as $F_s = 5\Phi_s \rho c_s^3$, where ϕ_s is the saturation constant (presumably $\lesssim 1$), ρ is the gas mass density and c_s is its sound speed. With this prescription, the largest achievable flux is approximated as the product of the thermal energy density in electrons times their characteristic thermal speed (assuming a thermal distribution and equal electron and ion temperatures; see Cowie and McKee 1977 for details). Because it is a saturated flux, it no longer explicitly depends on the magnitude of the temperature gradient but only on the direction of this gradient. Heat will flow outward in a hot accretion flow with a near-virial temperature profile, hence the positive sign adopted for F_s.

With this simple formulation for conduction, the steady-state 1-temperature self-similar ADAF solution of Narayan & Yi (1994) can be generalized to include the divergence of the saturated conduction flux in the entropy-energy equation.[2] Adopting the same geometry and notation as Narayan & Yi (1994), we consider a "visco-turbulent", differentially-rotating hot flow which satisfies the following height-integrated equations for the conservation of momentum and energy

$$v\frac{dv}{dR} = R(\Omega^2 - \Omega_K^2) - \frac{1}{\rho}\frac{d}{dR}(\rho c_s^2), \tag{1}$$

$$v\frac{d(\Omega R^2)}{dR} = \frac{1}{\rho R H}\frac{d}{dR}\left(\frac{\alpha\rho c_s^2 R^3 H}{\Omega_K}\frac{d\Omega}{dR}\right), \tag{2}$$

$$2H\rho v T\frac{ds}{dR} = f\frac{2\alpha\rho c_s^2 R^2 H}{\Omega_K}\left(\frac{d\Omega}{dR}\right)^2 - \frac{2H}{R^2}\frac{d}{dR}(R^2 F_s), \tag{3}$$

supplemented by the continuity equation, which is rewritten as $\dot{M} = -4\pi R H v\rho$. The last term in Eq (3) is the additional divergence of the saturated conduction flux. In these equations, R is the cylindrical radius, Ω_K is the Keplerian angular velocity around the central gravity-dominating mass, v is the gas radial (in)flow speed, Ω is its angular velocity, ρ is the mass density, c_s is the isothermal sound speed, s is the gas specific entropy, T is its temperature, $H = Rc_s/v_K$ is the flow vertical height and $f \leq 1$ is the advection parameter ($f = 1$ for negligible gas cooling). A Shakura-Sunyaev prescription has been used to capture the "visco-turbulent" nature of the flow, with an equivalent kinematic viscosity coefficient $\nu = \alpha c_s^2/\Omega_K$, where α is the traditional viscosity parameter.

We look for solutions to these equations of the form

$$\rho = \rho_0 R^{-3/2},\ v = v_0 R^{-1/2},\ \Omega = \Omega_0 R^{-3/2},\ c_s^2 = c_{s0}^2 R^{-1}, \tag{4}$$

and use the notation $v_K = v_{K0}R^{-1/2} = R\Omega_K = R\Omega_{K0}R^{-3/2}$. We find that the three unknowns v_0, Ω_0 and c_{s0} satisfy the relations

$$\frac{1}{2}v_0^2 - \Omega_{K0}^2 + \Omega_0^2 + \frac{5}{2}c_{s0}^2 = 0, \tag{5}$$

$$v_0 = -\frac{3\alpha}{2}\frac{c_{s0}^2}{v_{K0}}, \tag{6}$$

$$\frac{9\alpha^2}{8}c_{s0}^4 + \left(\frac{5}{2} + \frac{5/3-\gamma}{f(\gamma-1)}\right)c_{s0}^2 v_{K0}^2 - \frac{20\phi_s}{9\alpha f}c_{s0}v_{K0}^3 - v_{K0}^4 = 0, \tag{7}$$

while the density scale, ρ_0, is fixed by the accretion rate $\dot{M}$.

Contrary to the original ADAF solution, the energy relation (Eq. [7]) is not a quartic in c_{s0}/v_{K0} because of the extra saturated conduction term. We solve this fourth order polynomial with a standard numerical technique (Press et al. 1992) and discard the two imaginary roots as well as the real root with negative c_{s0}. While in the original ADAF solution the two real roots for c_{s0} were degenerate (in the sense that only c_s^2 mattered), this degeneracy is lifted up with saturated thermal conduction. The root with negative c_{s0} is physically unacceptable because it leads to a conduction flux going up the temperature gradient. Note that we have focused on accretion solutions ($v_0 < 0$) with $1 < \gamma < 5/3$ in our search, but by analogy with Narayan & Yi (1994), one also expect rotating wind solutions ($v_0 > 0$) with saturated thermal conduction to exist for values of $\gamma > 5/3$.

The main features of the new self-similar solutions with saturated conduction are illustrated in Fig. 1, as a function of the saturation constant, Φ_s. We are showing results for two distinct solutions here, with $\gamma = 1.5$ and $\gamma = 1.1$, and have fixed $f = 1$ and $\alpha = 0.2$ in both cases for definiteness. Standard ADAF solutions are recovered at small Φ_s values. As Φ_s approaches unity, however, the solutions deviate substantially from the standard ADAF. The sound speed, c_s, and the radial inflow speed, $-v_r$, both increase with the magnitude of conduction, while the squared angular velocity, Ω^2, decreases.

[2] Honma (1996) and Manmoto et al. (2000) have investigated the role of "turbulent" heat transport in ADAF-like flows. By relying on a saturated form of "microscopic" thermal conduction which is physically well-motivated, our analysis differs substantially from theirs. Saturation is key to a self-similar scaling, which would not obtain for a standard Spitzer conduction law.

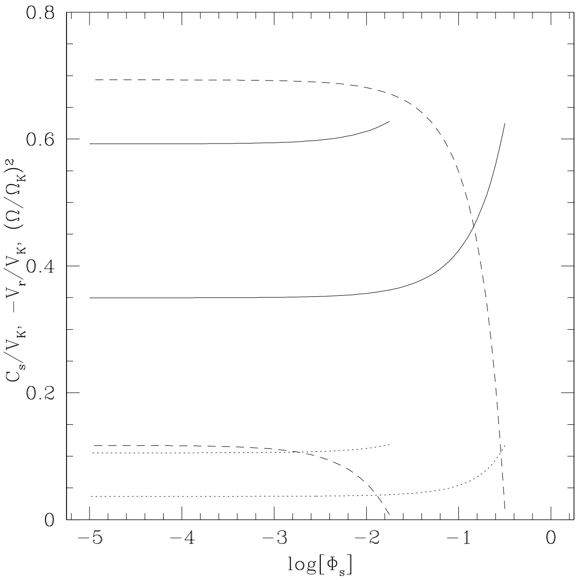

FIG. 1.— Changes in the sound speed (c_s/v_K, solid lines), angular rotation velocity (Ω^2/Ω_K^2, dashed lines) and radial inflow speed ($-v_r/v_K$, dotted lines) for two distinct solutions, as a function of the saturation constant, Φ_s. At low Φ_s values, these solutions match a regular ADAF. The solution extending up to $\log\Phi_s \simeq -0.5$ has a gas adiabatic index $\gamma = 1.1$, while the solution extending up to $\log\Phi_s \simeq -1.9$ has $\gamma = 1.5$. Both solutions have $\alpha = 0.2$ (viscosity parameter) and $f = 1$ (advection parameter).

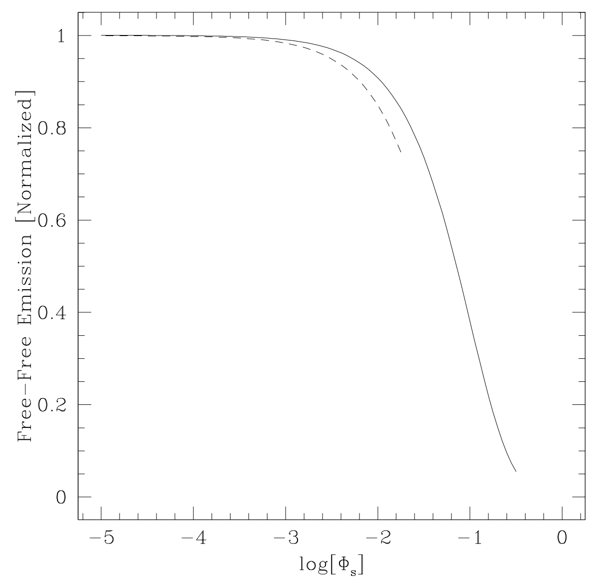

FIG. 2.— Drop in the level of free-free emission, relative to the standard ADAF value, as a function of the saturation constant, Φ_s, for the same two solutions as in Fig. 1 and a fixed accretion rate, $\dot{M}$.

Solutions still exist when Ω^2 becomes negative but they are obviously physically unacceptable. The solution with $\gamma = 1.5$ reaches the non-rotating limit at $\log\Phi_s \simeq -1.9$, while the solution with $\gamma = 1.1$ reaches this limit for $\log\Phi_s \simeq -0.5$. Both solutions have a sub-virial temperature ($c_s < v_k$) and a subsonic radial inflow speed ($-v_r < c_s$), which appear to be rather general properties. Supersonic solutions may exist for large α values when $\gamma \to 5/3$.

As the level of saturated conduction is increased, more and more heat flows from the hotter, inner regions, resulting in a local increase of the gas temperature relative to the original ADAF solution. Simultaneously, the gas adjusts its angular velocity (which reduces the level of viscous dissipation) and increases its inflow speed to conserve its momentum balance (increasing at the same time the level of advection). One can show from the energy equation that $\Omega^2 \propto (1 - |q_{\rm cond}/q_{\rm adv}|)$, where $q_{\rm cond}$ and $q_{\rm adv}$ are the magnitudes of the conduction and advection terms: solutions cease to exist once advection becomes unable to balance the heating due to saturated conduction. Solutions with $\Omega^2 = 0$ can be seen as Bondi-like with saturated conduction.

We have found that variations in the advection parameter, f, have little effect on the solutions, as long as $f \lesssim 1$. The breakdown of the solutions (when $\Omega^2 \to 0$) occurs at lower values of the saturation constant Φ_s for smaller α and larger γ values. While this breakdown may seem problematic, because it occurs for smallish values of Φ_s when $\gamma \lesssim 5/3$ (Fig. 1), it may well turn out to be a simple pathological feature of the 1D height-integrated equations we solved. By analogy with the results of Narayan & Yi (1995a) on convection in 2D self-similar ADAF solutions, we might expect conduction to dominate over advection only in the polar regions of the flow, in which case it may be possible to extend the solutions to larger Φ_s values (and perhaps launch outflows).

From the solution scalings, it is possible to calculate the modified level of emission expected relative to a standard ADAF. Assuming that free-free emission is the dominant mechanism, it scales as $\rho^2 T^{1/2} \propto (c_{s0} v_0^2)^{-1}$ for a given $\dot{M}$. Changes in this quantity as a function of the saturation constant, Φ_s, for the two same specific solutions of Fig. 1, are shown in Fig. 2 (normalized to the low-conduction=ADAF values). The emission is effectively reduced, potentially by a large factor (specifically by ~ 20 in our $\gamma = 1.1$ model). Even though conduction heats up the gas locally, the reduced density resulting from the larger inflow speed dominates, leading to a net decrease in the expected level of free-free emission. Clearly, by reducing the radiative efficiency at a given $\dot{M}$, saturated conduction could help explain the phenomenology of dim accreting black holes. However, work beyond these idealized self-similar solutions will be required before we can elucidate the role of conduction in hot accretion flows.

4. DISCUSSION AND CONCLUSION

The possibility that tangled magnetic fields strongly limit the efficiency of thermal conduction in a hot flow remains a major theoretical uncertainty. In a sense, we have accounted for this possibility by allowing the value of the saturation constant, Φ_s, to vary in our solutions. In recent years, a similar issue has been discussed in the cooling flow context: several studies have concluded that magnetic fields would probably not strongly limit thermal conduction (e.g. Narayan & Medvedev 2001; Gruzinov 2002; see also Chandran & Cowley 1998; Chandran et al. 1999), but it remains somewhat of an open issue. Explicit numerical simulations with anisotropic conduction along fields in an MHD turbulent medium could greatly help in settling this issue (see, e.g., Cho & Lazarian 2004).

An important limitation of the self-similar solutions with saturated conduction is their 1-temperature structure. Narayan & Yi (1995b) have shown how the 2-temperature

property of the hot flow is crucial to obtain self-consistent solutions with realistic cooling properties. The decoupling of the electron and ion temperatures would modify the role of saturated conduction in the inner regions of the hot flow, an effect which is not captured by our solutions. In this situation, the ions may be insulated from electron conduction. In addition, self-similarity is broken in the 2-temperature regime and the electron temperature profile flattens to a sub-virial slope (e.g. Narayan, Mahadevan & Quataert 1998). While this suggests a reduced role for conduction, the break-down of self-similarity also implies that the heating of the outer flow regions must be done at the expense of the hottest inner regions. Given the strong dependence on temperature of cyclo-synchrotron emission, even a moderate drop in temperature in the hottest regions could lead to a significant reduction in the radio emission properties of the flow. More detailed models are needed to address these issues and determine quantitatively by how much conduction can contributes to the low-radiative efficiency of dim accreting black holes.

In the last few years, much emphasis has been put on reducing the rate at which gas in a hot flow is accreted by the black hole, either by invoking a positive Bernoulli constant to justify the existence of powerful outflows (Blandford & Begelman 1999) or by invoking a net reduction in the accretion rate due to convection (Narayan et al. 2000; Quataert & Gruzinov 2000). The presence of strong conduction in the flow may pose a challenge to both of these scenarios: if conduction is important on all scales of interest in the hot flow (because of the large mean free paths), the Bernoulli constant[3] becomes irrelevant (as it is a property of adiabatic flows) and convection may be suppressed when any displaced fluid element efficiently reaches thermal equilibrium with its environment through conduction (but see Balbus 2001). Obviously, this is not to say that outflows are not expected from hot accretion onto black holes, since different configurations than the ones described by idealized steady-state analytical solutions are certainly possible (as shown explicitly by global turbulent MHD simulations).

If conduction is indeed important for hot accretion onto black holes, it could also have other interesting consequences besides the ones illustrated in our specific solutions. For instance, if the hot flow inside of the capture radius around a black hole is able to heat up the ambient gas outside of that capture radius, the Bondi rate of gas capture may be effectively reduced. While Gruzinov (1998) invoked "turbulent" heat flows to achieve this, the weakly-collisional conditions discussed in §2 for several galactic nuclei suggest that this could be done by "microscopic" conduction itself. A reduced rate of gas capture below the Bondi value could go a long way in explaining the low luminosity of these accreting black holes, but here again detailed models are required to address this issue quantitatively. Conduction could also have important consequences for the dynamical structure of hot flows, as argued recently by Balbus (2001) in a stability analysis in the presence of anisotropic conduction. In addition, in a weakly-collisional hot flow, it is in principle possible for the "microscopic" viscous stress tensor to contribute significantly to the overall flow dynamics, along with the turbulent component usually emphasized in accretion theory (e.g. Quataert et al. 2002).

Finally, conduction may offer a solution to the disk evaporation problem. Around both stellar-mass and supermassive black holes, there is evidence for inner truncation of thin accretion disks (e.g. Esin et al. 2001; Quataert et al. 1999). The process which "evaporates" the inner disks into a hot flow is not well understood, however. A simple calculation shows that, independently of the mass of and distance from the black hole, the saturated conduction flux ($\sim \rho c_s^3$) available to heat up a thin disk embedded in a hot flow (using ADAF ρ and c_s scalings for simplicity; Narayan et al. 1998) is within $\sim \alpha^{-1}$ of the traditional viscous dissipation rate per unit area of the disk (for a same $\dot{M}$ in both components). This suggests that saturated conduction is energetically capable of contributing to the evaporation of thin disks in both classes of systems.

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[3] Satisfying the same Eq (5) as ADAFs (Narayan & Yi 1994), solutions with saturated conduction have formally larger positive Bernoulli constant because of an increased temperature and reduced angular velocity.

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